Technical Notes

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Some Iterative Schemes for Transonic Potential Flows

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Introduction

THE study of numerical solutions for transonic potential flows has received considerable attention recently. The standard procedure based on the successive line overrelaxation (SLOR) method is reliable but rather inefficient in the sense that even with an optimal relaxation parameter it suffers from slow convergence. Alternative methods such as the ZEBRA scheme[1] and the approximate factorization (AF) method[2] have been suggested. These methods provide faster convergence rates than the SLOR method if the corresponding iteration parameters are properly determined. Motivated by the difficulties in choosing optimal parameters a priori, it is of strong interest to develop an efficient and reliable method which does not require an estimation of any iteration parameter.

The conjugate gradient (CG) method has been widely used for solving large sparse symmetric and positive-definite linear equations. Note that, not only does this method provide a fast rate of convergence, it also does not require any knowledge of iteration parameters. Khosla and Rubin[3] adapted this method and presented results for subsonic flows over an airfoil. For transonic flow calculations, Wong and Hafez[4] suggested a combination of the SLOR and the preconditioned CG methods.

To avoid a combined iteration, the minimal residual (MR) method will be studied herein. This method is closely related to the CG method, and it can be regarded as a first-order gradient method. The primary advantage of this method is that it is applicable to symmetric and nonsymmetric matrices.

Problem Formulation

The full-potential equation expressed in conservation form is written as

$$(\rho\phi_x)_x + (\rho\phi_y)_y = 0 \qquad (1)$$

where

$$\rho = \left[1 - \frac{\gamma - 1}{\gamma + 1}(\phi_x^2 + \phi_y^2)\right]^{1/(\gamma - 1)}$$

and ϕ is the velocity potential, ρ the density, and γ the ratio of specific heats. For complex geometries it is advantageous to transform Eq. (1) from the Cartesian coordinates into the computational domain in a rectangle

$$(\rho U/J)_\xi + (\rho V/J)_\eta = 0 \qquad (2)$$

where J is the Jacobian of the transformation, and U and V are the contravariant velocity components along the transformed coordinates ξ and η. The fluid density is modified such that

$$\rho \leftarrow (\rho - \mu\rho_\xi \Delta\xi) \qquad (3)$$

so that an artificial viscosity is introduced. Here μ is a switching function zero in subsonic regions and nonzero in supersonic regions, ρ_ξ is the density gradient in the streamwise direction.

Equation (2) is a nonlinear mixed elliptic-hyperbolic partial differential equation. However, assuming ρ is given, a central finite difference approximation to Eq. (2) gives

$$L\phi = 0 \qquad (4)$$

where L is a large sparse matrix operator.

Iterative Schemes

Approximate Factorization (AF) Scheme

The AF scheme is given by
Step 1:

$$(\sigma - \vec{\partial}_\eta B_j) f_{i,j}^n = \alpha\omega L\phi_{i,j}^n$$

Step 2: (5)

$$(\alpha\overleftarrow{\partial}_\eta \pm \alpha\beta\overleftrightarrow{\partial}_\xi - \overleftarrow{\partial}_\xi B_i \vec{\partial}_\xi)\delta\phi_{i,j}^n = f_{i,j}^n$$

where B_i and B_j are the metric quantities, α an acceleration parameter, $f_{i,j}$ an intermediate result, $\delta\phi_{i,j}$ the correction vector, and $L\phi_{i,j}$ the residual. Note that the difference approximation is split between the two steps in the η direction. This generates a $\phi_{\eta t}$-type term. A $\phi_{\xi t}$-type term has been added in step 2. The parameter β is updated depending on the changes in the residual.

ZEBRA Scheme

The SLOR scheme is given by

$$\sigma_u B_{i-1}\delta\phi_{i-1,j} + \sigma_1 B_i \delta\phi_{i+1,j} + B_{j-1}\delta\phi_{i,j-1}$$
$$- [(B_{i-1} + B_i) + (B_{j-1} + B_j)]\delta\phi_{i,j} + B_j\delta\phi_{i,j+1} = -L\phi_{i,j}^n$$
$$\phi_{i,j}^{n+1} = \phi_{i,j}^n + \omega\delta\phi_{i,j} \qquad (6)$$

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where ω is the relaxation parameter, on the upper surface $\sigma_u = 1.0$, $\sigma_l = 0.0$, and on the lower surface $\sigma_u = 0.0$, $\sigma_l = 1.0$. A tridiagonal solver is used for each line ($\xi = \text{const}$). Notice that marching is always with the flow direction. The ZEBRA scheme is based essentially on the SLOR with red and black ordering.

Minimal Residual (MR) Scheme

The solution of Eq. (4) can be obtained by solving

$$M\delta\phi^n = -L\phi^n \tag{7}$$

where $\delta\phi^n = \phi^{n+1} - \phi^n$, $L\phi^n$ is the residual vector at the nth iteration, and M a matrix operator. Here M is chosen from the operator L by ignoring the skewness effect due to the grid transformation. Thus M is an approximation of L. Note that if M is the Jacobian of L, then Eq. (7) is a Newton iterative method. Therefore, the procedure considered here can be regarded as a Newton-like iterative scheme. However, it will converge only for subsonic flows. For mixed subsonic-supersonic flow calculations M must be modified so that a $\phi_{\xi t}$ term is included

$$M \leftarrow (M \pm \mu\beta \overleftrightarrow{\partial}_\xi) \tag{8}$$

where μ is the switching function as defined previously, and β determines the amount of $\phi_{\xi t}$ introduced.

The MR method is then applied to solve Eq. (7), and a preconditioning matrix operator C is introduced to accelerate the rate of convergence. The preconditioned MR algorithm is as follows.

Set $r_0 = L\phi_0 - M\delta\phi_0$, solve $Cp_0 = r_0$, then for $k = 0,1,2,\ldots,\bar{k}$,

$$\begin{aligned} \delta\phi_{k+1} &= \delta\phi_k + \alpha_k p_k \\ r_{k+1} &= r_k - \alpha_k M p_k \\ \alpha_k &= (r_k, Mp_k)/(Mp_k, Mp_k) \\ Cp_{k+1} &= r_{k+1} \end{aligned} \tag{9}$$

The main computational requirements are one matrix-vector multiplication Mp and one solution of $Cp = r$ for each k. The preconditioning matrix C is based on an incomplete LU factorization of M, which can be factorized into a product of sparse lower and upper triangular matrices. Thus the solution of $Cp = r$ is obtained efficiently by simple backward and forward substitutions.

Numerical Results

Transonic potential flows around an NACA 0012 airfoil are calculated for different Mach numbers and angles of attack. The three different iterative schemes described in the previous section were implemented based on the TAIR[5] code. The convergence histories for nonlifting and lifting airfoil calculations are given in Figs. 1 and 2, where the grid size is 149×30.

The TAIR code based on an AF scheme is reliable and very fast. There are three parameters in the calculations (α_l, α_h, and ω) and the rate of convergence is not very sensitive to small variations of these parameters. It is obvious that the performance of AF is satisfactory. However, there are some questions about the approximate factorization involved, and it is not clear how the error terms affect the convergence of transonic calculations. Another point of concern is the treatment of the boundary conditions for the intermediate variable. It is handled in the code in an empirical way. The parameter α which is cyclically varied between α_l and α_h is restricted to never fall below AFAC$*\alpha_l$ in the neighborhood of the airfoil boundary, where AFAC is a multiplier parameter used in the TAIR code, and its default value is 14.0.

The ZEBRA scheme is simple and reliable. The convergence, however, depends on a relaxation parameter ω, which is chosen by numerical experiments. The CPU time of ZEBRA and AF iterations (using the same grid) is almost the same. An improved version of this scheme can be found in Ref. 6.

The main advantage of the MR algorithm is that no relaxation parameters ω or α's as in other schemes are required. The convergence histories for the MR method are smooth compared to AF and ZEBRA tested here, and the efficiency of this method depends upon the preconditioning operator C. The method was first implemented using a simple incomplete LU factorization, in which L and U have three nonzero diagonals.

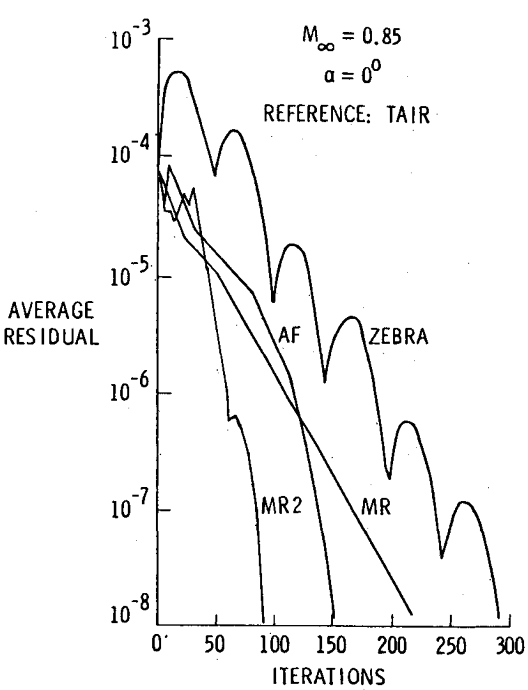

Fig. 1 Comparison of convergence histories; NACA 0012 airfoil, $M_\infty = 0.85$, $\alpha = 0$ deg.

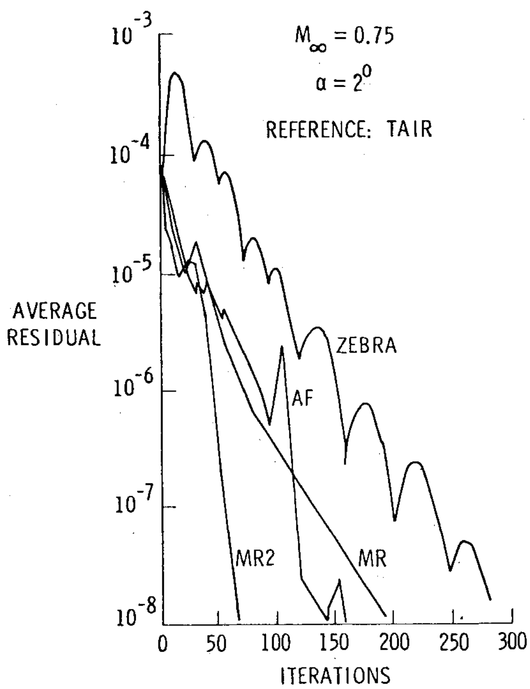

Fig. 2 Comparison of convergence histories; NACA 0012 airfoil, $M_\infty = 0.75$, $\alpha = 2$ deg.

These results were denoted in Figs. 1 and 2 by MR and indicated that not only were more iterations needed, but the CPU time per iteration also took about three times that required for the AF or ZEBRA scheme. Consequently, it was not competitive with the AF scheme. However, by using a more accurate LU factorization (i.e., by introducing more nonzero diagonals into the L and U matrices), a faster convergence rate was achieved. Since the number of iterations for both outer and inner iterations is reduced substantially, the performance of the new MR method (denoted by MR2 in Figs. 1 and 2) is more efficient than the old MR method. A detailed implementation of the improved MR method (including the construction for the preconditioning operator C), and comparisons with the AF scheme for lifting and nonlifting calculations, have been reported by Wong.[7]

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Refinement of an "Alternate" Method for Measuring Heating Rates in Hypersonic Wind Tunnels

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Nomenclature

c = specific heat, J/kg-K
C_h = heat-transfer coefficient, $\dot{q}_{t,2}/(T_{t,2}-T_s)$, W/m^2-K
k = thermal conductivity, W/m-K
M = Mach number
p = pressure, N/m^2
$\dot{q}$ = heat-transfer rate, W/m^2
r_n = nose radius, m
R = unit Reynolds number, $\rho U/\mu$, m^{-1}
t = time, s
T = temperature, K
U = velocity, m/s
β = thermal product, $(\rho ck)^{1/2}$, W-s$^{1/2}$/m^2-K
μ = coefficient of viscosity, N-s/m^2
ρ = density, kg/m^3

Subscripts

s = surface or substrate
$t,1$ = reservoir stagnation conditions
$t,2$ = stagnation conditions behind normal shock
2 = static conditions immediately behind normal shock
0 = ambient or initial ($t \leq 0$)
∞ = freestream

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Introduction

CONVECTIVE heating for proposed Earth or planetary entry vehicles is generally inferred from model tests in hypersonic facilities and flowfield computer codes that often are verified with these or similar heat-transfer measurements. Such measurements are relatively straightforward for models of a simple shape, but difficulties arise for complex models characterized by surfaces with small radii of curvature (e.g., wing leading edge). Accurate measurement of surface heating using conventional transient calorimeter techniques becomes quite challenging for complex models and often is impossible. For this and other reasons, an alternative method for measuring detailed heating distributions on models in conventional-type hypersonic wind tunnels was examined at the Langley Research Center several years ago.[1] Borrowing from technology developed for hypervelocity impulse facilities having extremely short run times (< 10 ms), thin-film resistance gages were successfully tested in hypersonic wind tunnels at the most hostile environment generated in each tunnel. One significant difference between these gages and previous thin-film gages used in impulse facilities was the substrate material. A glass ceramic referred to as MACOR (trademark of Corning Glass Works) was used in lieu of quartz or Pyrex.[2] Because the thermal properties of MACOR are similar to quartz and Pyrex, gage performance is similar; however, unlike quartz and Pyrex which must be ground, MACOR may be machined with conventional metal cutting tools and techniques. Thus, complex surfaces may be machined from MACOR, polished, and miniature thin-film elements sputtered or painted on the surface at a cost substantially less than quartz or Pyrex.

The advantages, disadvantages, limitations, and uncertainties of using thin-film gages in conventional hypersonic wind tunnels having longer run times than impulse facilities by several orders of magnitude are discussed in Refs. 1 and 3. The purpose of this Note is to present recent results that resolve the principal uncertainty associated with the use of thin-film gages on MACOR substrates—the uncertainty in the thermal properties of MACOR and the variation of these properties with temperature. These results are based on stagnation-point heat-transfer rates measured on small hemispheres in hypersonic wind tunnels. Along with providing information on MACOR properties, they also illustrate the relatively large influence of shock strength (normal shock density ratio) on stagnation point heating at low Reynolds numbers.

Apparatus and Tests

The present study was performed in the Langley 31 in. Mach 10 Tunnel[4] (formerly known as the Continuous Flow Hypersonic Tunnel), and the Langley Hypersonic CF_4 Tunnel,[4] a Mach 6 facility that uses Freon 14 to generate a normal